Active Sampling for Accelerated MRI with Low-Rank Tensors

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Motivation: Accelerating MR Imaging

Imaging speed is important in MR imaging. It is highly desired to design efficient sampling patterns for accelerating imaging.

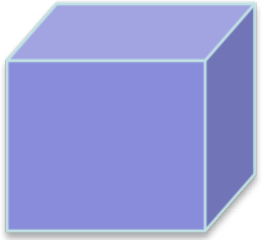
Tensor, as a natural generalization of matrix, becomes more and more popular recently. But its sampling design is less investigated.

Tensor background

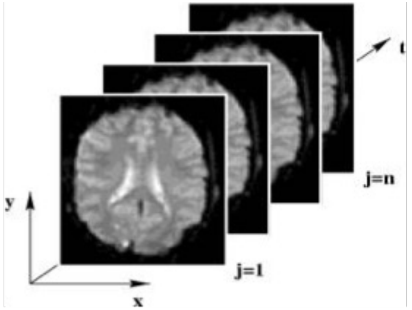
- matrix: 2-D data array

$$\mathbf{A} = [a_{i_1 i_2}] \in \mathbb{R}^{n_1 \times n_2}$$

- 3-D tensor



$$\mathcal{A} = [a_{i_1 i_2 i_3}] \in \mathbb{R}^{n_1 \times n_2 \times n_3}$$



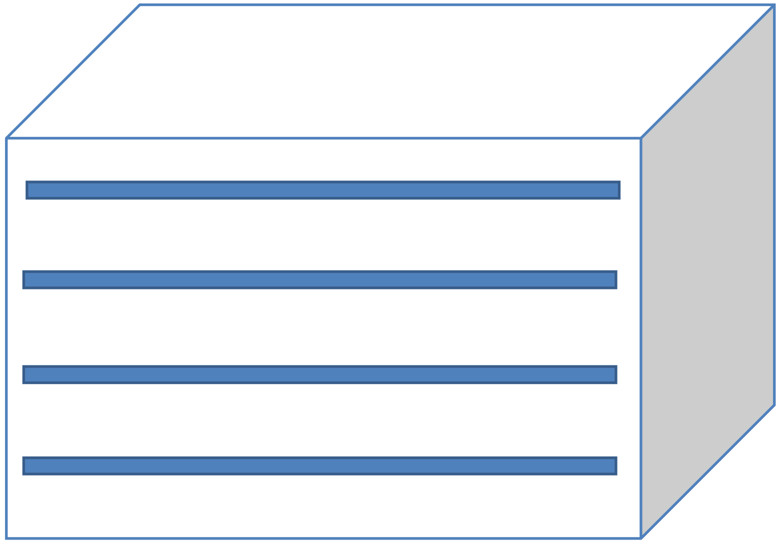
- General case: d-dimensional tensor

$$\mathcal{A} = [a_{i_1 \cdots i_d}] \in \mathbb{R}^{n_1 \times \cdots \times n_d}$$

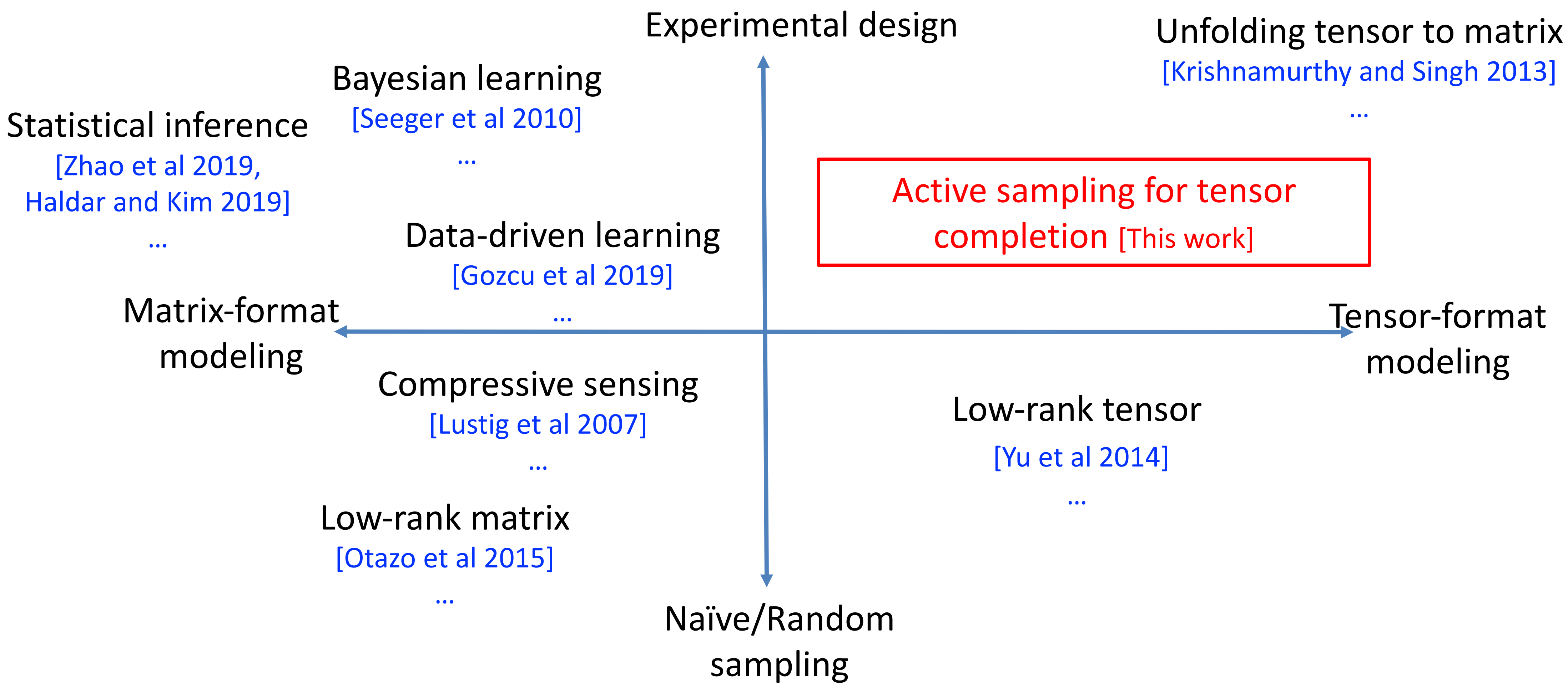
Challenges in tensor-format sampling design

Lots of matrix-format sampling method naturally apply by unfolding a tensor to a matrix, but it hurts the structure information

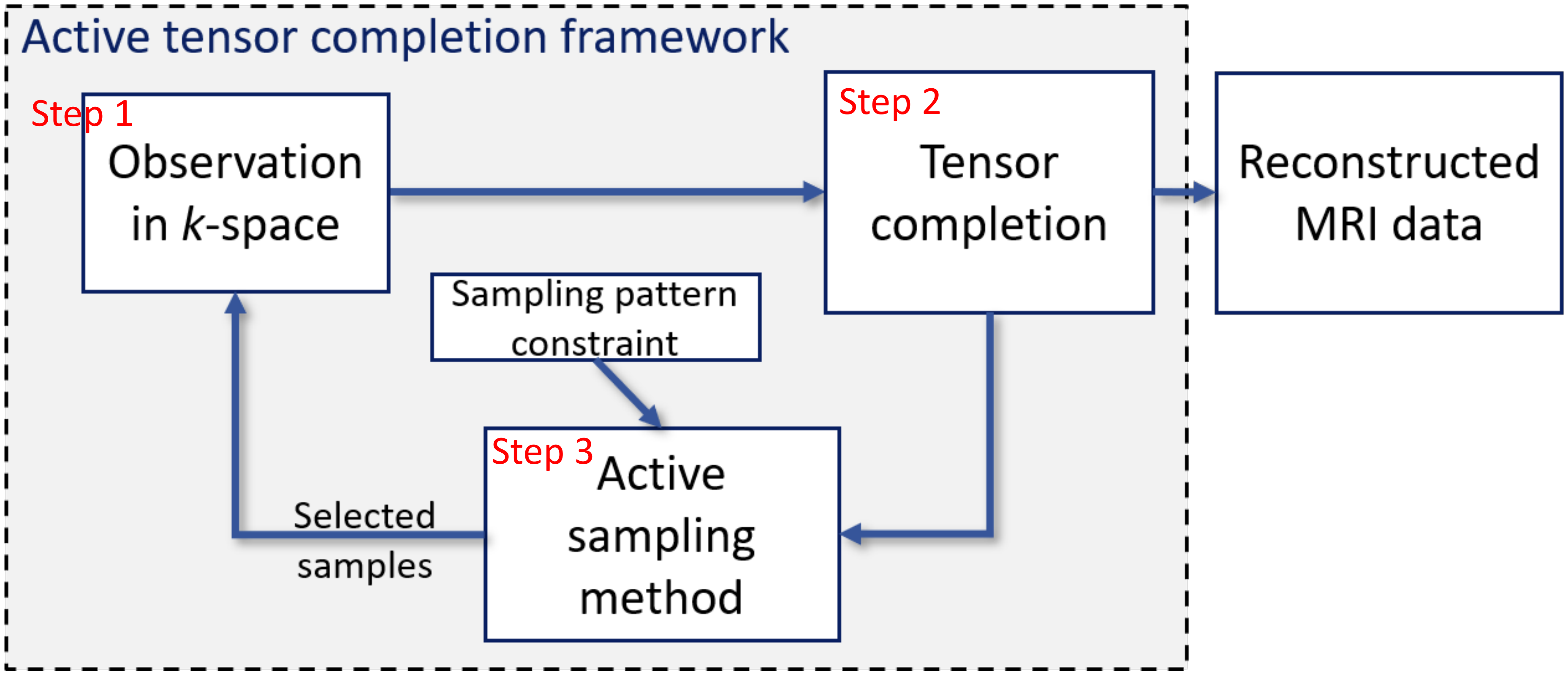
Pattern constraints, like Cartesian sampling, need to be satisfied



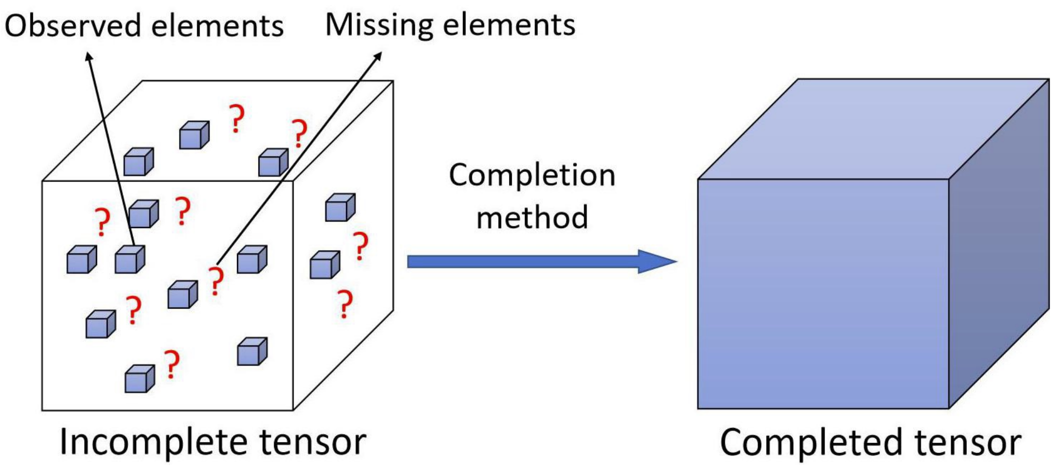
Existing works



Proposed method overview



Tensor completion model



[Image credit to Zheng-bin]

Reconstruct the k-space data by assuming the low-rankness of its structure..

$$\begin{aligned} \min_{\{\mathbf{X}_n\}_{n=1}^{N}, \mathcal{M}} \quad & \sum_{n=1}^{N} \|\mathbf{X}_n\|_* \\ \text{s.t.} \quad & \mathbf{X}_n = \mathbf{M}_{(n)},\ n = 1, 2, \ldots, N, \\ & \mathcal{M}_\Omega = \mathcal{T}_\Omega, \end{aligned}$$

Can be solved by many alternative solvers. Output reconstruction from N modes

Query-by-Committee method

QBC is a traditional active learning model in ML: A committee of models will decide which sample to be added

- A committee of models: different mode output serve as the committee
- Sample quality measurement. The measurement is desired to be adaptable to different sampling patterns.

Sample quality measure

Predictive variance

Measure output difference among different modes, capture the uncertainty towards reconstruction model

Leverage score

Given a matrix A=USV’, leverage score measures coherence of a row/column with a coordinate direction.

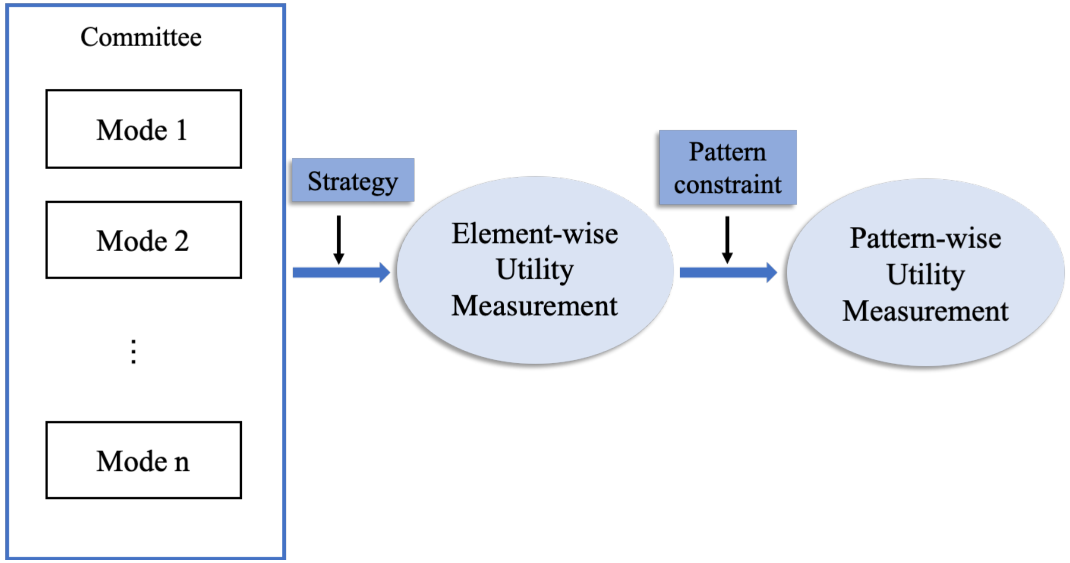
$$\ell(i) := \tfrac{N_1}{R}\left\|\mathbf{U}^T\mathbf{e}_i\right\|_2^2, \quad i = 1, 2, \ldots, N_1,$$
$$\mathbf{r}(j) := \tfrac{N_2}{R}\left\|\mathbf{V}^T\mathbf{e}_j\right\|_2^2, \quad j = 1, 2, \ldots, N_2.$$

The dot product of row & column score represents the importance of an element.

Sampling pattern constraints

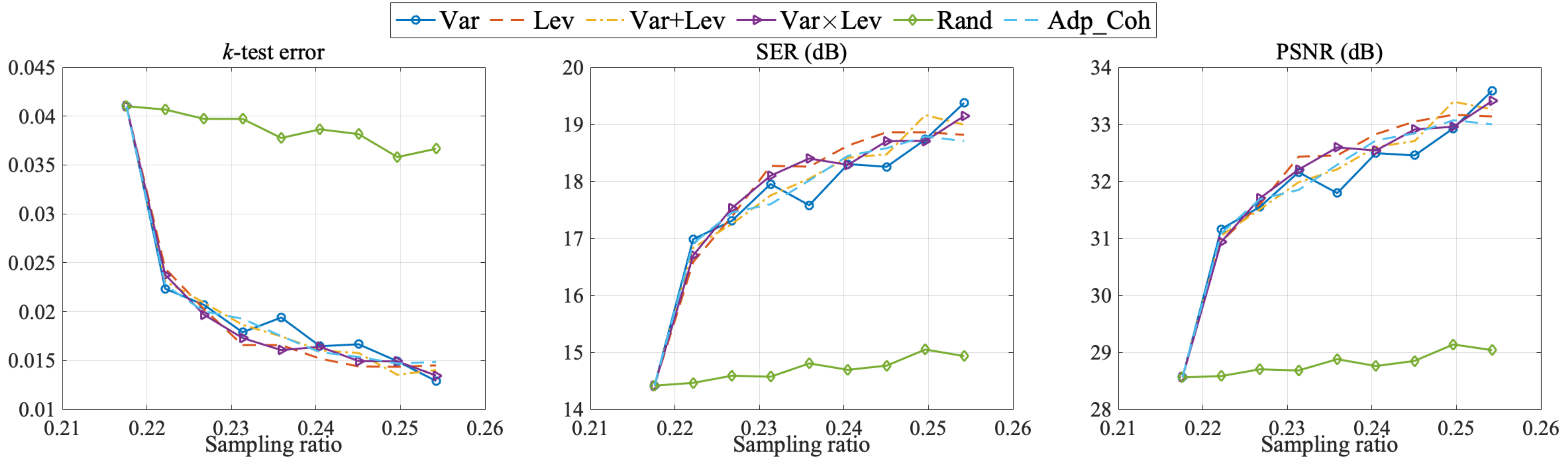
Some pattern constraints, like Cartesian sampling (select a fiber of the tensor), need to be satisfied in the sample selection.

Fortunately, our sample quality measure are all element-wise. It is straightforward to construct a pattern satisfying arbitrary constraint.



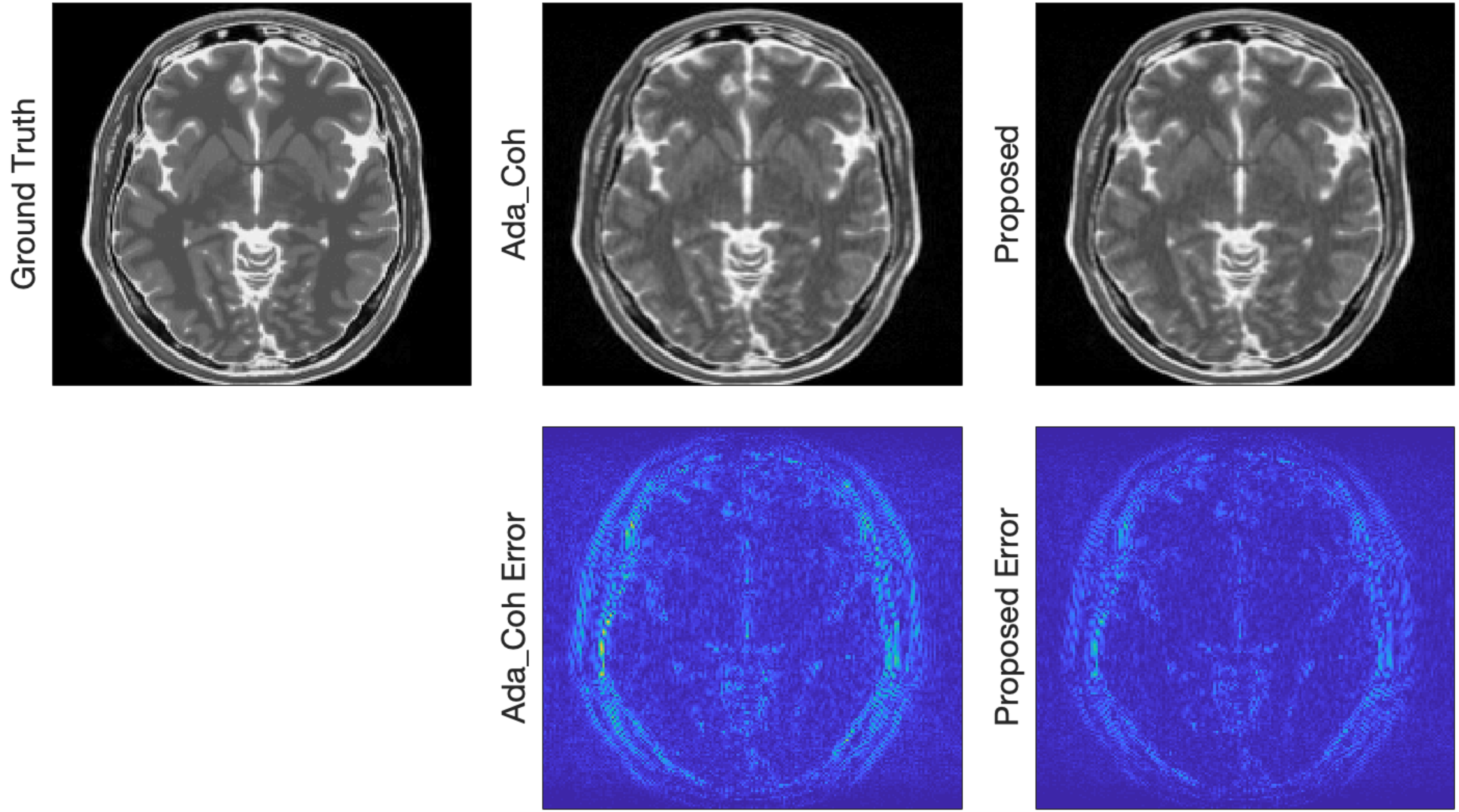
Numerical results

Reconstruct a 3D-brain image by the tensor completion model with query-by-committee with various measure.

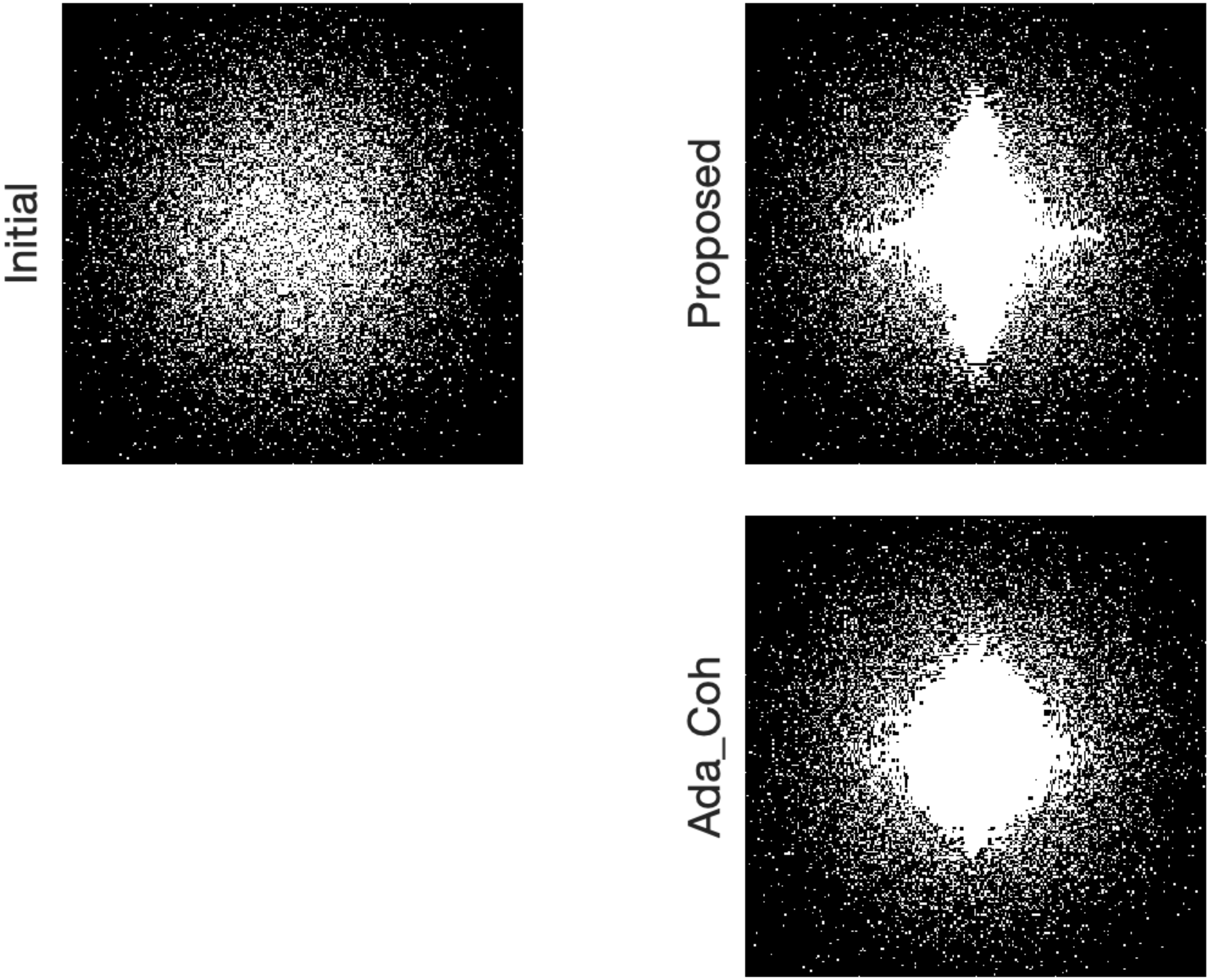


Adp_Coh [Krishnamurthy and Singh 2013] unfolds a tensor to a matrix and use matrix leverage score

Numerical results



Numerical results



Take-home message

We propose a query-by-committee based active learning method for tensor-based MR Imaging

- Select better samples for the tensor model by leveraging information from all modes
- Sampling pattern constraints are easy to satisfy
- The application is not limited to medical imaging

Future works

Online sampling: to incorporate the active sampling process into the tensor completion model

Verify on different kinds of realistic MRI data

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